1. **Calculate the day length and mean daily solar TOA insolation for the June 21 solstice at 65°N latitude. Assume the true anomaly is linearly proportional to time.**

The solar declination on the summer solstice is equal to the obliquity of the Earth, which is \( \delta = \epsilon = 23.5^{\circ} \). The day length is obtained from the solar declination and the latitude \( \phi = 65^{\circ} \):

\[
\cos H = -\tan \phi \tan \delta = -(0.435)(2.145) = -0.932 \quad H = 137^{\circ}(-43.0^{\circ} + 180^{\circ})
\]

Day length is \((24 \text{ hr})H/180^{\circ} = 18.3 \text{ hrs}\)

June 21 is about 169 days after the January 3 perihelion. With the linear in time assumption the true anomaly is

\[
\nu \approx 360(169/365.24) = 166.6^{\circ}
\]

The ratio of the Earth-Sun distance to the mean distance is then

\[
\frac{r}{r_0} \approx 1 - e \cos \nu = 1 - (0.017) \cos(166.6) = 1.0165
\]

The daily averaged solar flux is

\[
\bar{F} = \frac{S_0}{\pi} \left( \frac{r_0}{r} \right)^2 \left( H \sin \phi \sin \delta + \sin H \cos \phi \cos \delta \right)
\]

In radians \( H = 2.39 \), so the TOA daily mean solar flux is

\[
\bar{F} = \frac{S_0}{\pi} (1.0165)^{-2}[2.39 \sin 65 \sin 23.5 + \sin 137 \cos 65 \cos 23.5]
\]

\[
\bar{F} = (1366 \text{ W/m}^2)(0.968)(1.128)/\pi = 475 \text{ W/m}^2
\]

2. **Calculate the mean daily solar insolation for the December 21 solstice at 65°S latitude. Compare with the result in 1.**

At the other solstice the solar declination is \( \delta = -23.5^{\circ} \). Since we are now using a latitude of \( \phi = -65^{\circ} \), the day length will be the same as in 1. The only thing that is different for the daily mean solar flux is the Earth-Sun distance.

December 13 is about 13 days before the January 3 perihelion. With the linear in time assumption the true anomaly is

\[
\nu \approx 360(-13/365.24) = -12.8^{\circ}
\]

The ratio of the Earth-Sun distance to the mean distance is then

\[
\frac{r}{r_0} \approx 1 - e \cos \nu = 1 - (0.017) \cos(-12.8) = 0.983
\]

The daily mean TOA solar flux is then

\[
\bar{F} = (1366 \text{ W/m}^2)(1.034)(1.128)/\pi = 507 \text{ W/m}^2
\]

This is 32 W/m² or about 7% higher than the situation for the other hemisphere and solstice.