Please show your work.

1. Below are two atmospheric spectra for the US standard atmosphere, one for transmission from space to 11 km and the other for transmission to sea level. Explain the large differences between the two spectra in terms of the distribution of absorbing gases. Label the shortwave and longwave portions. Label on the plots as many of the absorption features listed in the lecture notes as you can (e.g. CO$_2$ $\nu_2$).

2. The line spacing for the regular isotope of CO$_2$ ($\nu_2$ band center at 667.38 cm$^{-1}$) is 1.562 cm$^{-1}$. For the $^{16}$O-^{12}$C$-$^{18}$O isotope of CO$_2$ ($\nu_2$ band center at 662.37 cm$^{-1}$) the line spacing is 0.737 cm$^{-1}$. Explain this quantitatively in terms of the allowed rotational quantum numbers $J$ and the rotational constant $B$.

3. a) Show that the ratio of the Lorentz halfwidth to the Doppler halfwidth is approximately proportional to air density. Obtain an expression for the air density at which the Lorentz and Doppler halfwidths (at half max) are equal.

   b) Calculate the density, and find the approximate height from a standard atmosphere, where the Lorentz and Doppler halfwidths are the same for water vapor absorption lines at 183.3 GHz, 1560 cm$^{-1}$, and 1.37 $\mu$m. For each of these lines the Lorentz halfwidth at $T_0 = 296$ K and $p = 1.013 \times 10^5$ Pa is about $\alpha_0 = 0.10$ cm$^{-1}$.

4. a) Why does the 10 cm$^{-1}$ resolution atmospheric transmission (such as in question 1) increase going from the band center to the band edge of an absorption feature? Explain in terms of the physics of molecular absorption. You may wish to use the 15 $\mu$m CO$_2$ band as an example.

   b) The 708 to 724 cm$^{-1}$ transmission from 100 to 18 km is 0.765, while from 5 to 4 km it is only 0.425, even though these two paths contain virtually the same absorber amount $u$ of CO$_2$ ($\Delta p = 76$ mb in each case). Explain this difference in transmission in terms of the behavior of molecular absorption lines.
MODTRAN3 Transmission (US Standard Atmosphere)

10 cm\(^{-1}\) resolution

Transmission vs. Wavenumber (cm\(^{-1}\))